Mobility and Fiscal Imbalance

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Abstract
We study how labor mobility affects the optimal fiscal gap in a federation and the fiscal imbalances that arise because of productivity shocks. Fiscal imbalance – a deviation from the optimal fiscal gap – occurs when the second-best allocation of resources in a federation cannot be achieved because fiscal transfers do not or cannot undo fiscal externalities among regional and federal governments. Under reasonable circumstances, we find that labor mobility increases the optimal fiscal gap, that is, increases the transfers required to achieve the second-best optimum. In a decentralized federation, the optimal fiscal gap cannot be achieved. In the absence of labor mobility, vertical fiscal externalities will apply. Regional governments will overspend, which will induce the federal government to create a negative fiscal imbalance to contain the size of its tax rate, assuming it can commit to future transfers. If the federal government cannot commit, regions will overspend even more and federal transfers will be excessive, leading to a positive fiscal imbalance. In both cases, mobility of labor mitigates the fiscal imbalance by reducing the tendency of regions to overspend.

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1 Introduction

The assignment of expenditure and revenue-raising functions among governments in a federation is one of the classical problems of fiscal federalism, going back to the work of Musgrave (1959), Oates (1972) and McLure (1983). A key aspect of the assignment problem is finding the correct balance between expenditure and revenue-raising responsibilities on the one hand, and the system of federal-regional transfers used to reconcile them on the other. The term fiscal gap captures the idea that it may be preferable to decentralize expenditure decisions more than revenue-raising responsibilities, and to use transfers to facilitate these preferences. An optimal fiscal gap will be a set of transfers that just enables the two levels of government to pursue their fiscal responsibilities in an ideal way, for example, to achieve some second-best level of efficiency in the federation. Moreover, the optimal fiscal gap may incorporate the need for transfers to differ among regions because of some underlying heterogeneity in their ability to provide desirable levels of goods and services. The exact form of the optimal fiscal gap will differ according to the nature of the federation being studied. Factors such as the mobility of goods, factors and households, differences among regions, and the objectives of government will influence both the optimal extent of decentralization and the size of the fiscal gap across regions. The literature on the optimal fiscal gap is relatively small, but includes contributions by Gordon (1983), Dahlby (1996), Boadway and Keen (1996), Dahlby and Wilson (1994), Persson and Tabellini (1996a, 1996b) and Sato (2000).

Our concern is also with a related concept, that of fiscal imbalance. Loosely speaking, fiscal imbalance exists if the allocation of fiscal responsibilities between federal and regional governments on the one hand and across regional governments on the other are such that the optimal fiscal gap is not achieved. Equivalently, the size of federal-regional transfers does not coincide with the second-best optimal level. The problem of fiscal imbalance, and its relationship to the optimal fiscal gap have been prominent in the Canadian federation recently and has given rise to much policy debate. There, a fiscal imbalance between the federal government and the provinces was alleged to exist as a consequence of the federal government addressing their fiscal difficulties by preemptively reducing transfers to the provinces, while at the same time a horizontal imbalance existed across provinces.
as a result of natural resource shocks.\textsuperscript{1} One of the consequences of the latter has been a significant reallocation of workers to the provinces experiencing the resource shocks, a movement that is seemingly exacerbated by a failure of the fiscal transfer system to address the horizontal imbalance.

More generally, the idea of fiscal imbalance can apply whenever revenue-raising and the transfer system are misaligned with expenditure requirements. This can occur because of an asymmetric shock that the system of transfers is unable to ameliorate. It can also occur because of actions initiated by one level of government or the other in response to jurisdictional needs. Thus, a regional government may over-extend itself financially in anticipation of eliciting a federal transfer (or bailout): the so-called soft budget constraint.\textsuperscript{2} Alternatively, as in the Canadian case, the precipitous action of the federal government may leave one or more regions with fiscal deficiencies: an excessively hard budget constraint.

As these examples indicate, whether it is initiated by an external shock or by the actions of one level of government, the existence of fiscal imbalance typically presumes some interdependencies among governments in a federation. These can take various forms. There is an established literature on fiscal externalities, carefully surveyed by Dahlby (1996), that recognizes various ways in which decisions taken in one jurisdiction spill over into other jurisdictions. Regional government fiscal policies can directly affect residents of other jurisdictions, such as expenditures on public goods and services whose benefits transcend borders, or environmental policies that affect residents in neighboring jurisdictions. Alternatively, government policies in one region may indirectly affect residents elsewhere through the effect on government budgets in other regions, giving rise to so-called fiscal externalities. It is this sort of indirect interdependency that is our focus since it gives rise to fiscal imbalances.

In particular, there are two sorts of government-to-government fiscal interactions that can lead to fiscal externalities: horizontal and vertical. Horizontal fiscal externalities arise

\textsuperscript{1} The concept of fiscal imbalance in the Canadian context was introduced by the Commission on Fiscal Imbalance (2002), which was a study done for the Quebec government. The issue was thoroughly considered by the Council of the Federation (2006), which is a body representing the provinces.

\textsuperscript{2} See, for example, Wildasin (2004) and Vigneault (2007).
to the extent that fiscal policies in one region affect the fiscal capacities of other regional governments. This typically results either from mobility of tax bases or from cross-region ownership of tax bases. Thus, mobility of capital or cross-border purchases of products can give rise to tax competition, as a consequence of which tax rates are driven down below their efficient levels as regions compete to attract scarce tax bases.³ The case of labor mobility is less clear since, depending on the model, it may or may not be beneficial to use fiscal policies to attract more labor. Thus, in so-called Ricardian models of federalism with costless migration, where each region has a fixed factor (e.g., land) and each provides a regional public good, regional governments choose their tax rates and public goods supplies optimally under reasonable circumstances.⁴ This is because free migration makes the per capita utility in each region the same so that maximizing regional per capita utility is equivalent to maximizing national per capita utility, referred to as incentive equivalence by Myers and Papageorgiou (1993). On the other hand, mobility of low-income transfer recipients can give rise to similar tax competition effects as capital or product mobility. In this case, the interests of the regions and their governments typically coincide with those of the non-poor population who are immobile and who finance transfers to the poor based on altruistic motives.⁵

For future reference, one way of characterizing the consequences of tax competition is by its effect on the so-called marginal cost of public funds (MCPF). Since the region perceives a loss in tax base from an increase in its own tax rate as being a cost to itself (even though the cost is offset by a benefit to another region), it takes its MCPF as being higher than the ‘social’ MCPF. Cross-border ownership of tax bases can have an offsetting effect. If the tax bases are not highly mobile, regional governments will have an incentive to


⁴ See, for example, Boadway (1982) and Myers (1990). This issue is discussed in the recent survey of equalization by Boadway (2004).

⁵ Boskin (1973), Pauly (1973), and more recently Wildasin (1991) and Kessler, Lülfessman and Myers (2002), are examples of papers that study the inefficiencies arising from mobility of low-income households.
over-use source-based taxes if some of the owners are non-residents. That is, tax exporting will occur. In this case, the MCPF will be underestimated by regions.

Vertical fiscal externalities occur between levels of government, and apply even in the absence of mobility. The basic idea, which goes back to Johnson (1988), occurs in its starkest form when the federal and regional governments use a common tax base, provided it is variable in size. If an increase in a region’s tax rate reduces the size of the tax base, that will not only temper the increase in tax revenues the region will obtain, it will also reduce the tax revenue of the federal government. Regions will have little incentive to take this into account (unless the reduced federal taxes feed back into reduced transfers to the region), and will consequently tend to set too high a tax rate. Put differently, regions will underestimate their MCPF. Horizontal and vertical fiscal externalities due to tax competition thus tend to work in opposite directions. On balance, either could dominate.

There is ample empirical evidence for both horizontal and vertical fiscal externalities. The case of goods was first examined by Besley and Rosen (1998), who found evidence of vertical fiscal externalities in the case of cigarette and petroleum excise taxes in the USA. Devereux, Lockwood and Redoano (2007) extended the analysis to include both horizontal and vertical interaction, and found both to apply. Horizontal and vertical capital tax competition were estimated for Canada by Hayashi and Boadway (2001) and for the USA by Esteller-Moré and Solé-Ollé (2001). Brühlhart and Jametti (2006) studied horizontal and vertical externalities for the case of mobile labor in Switzerland.

The strength of the effect of fiscal externalities on regional behavior depends on the federal-regional transfer system. Indeed, the transfer system is a potentially important policy instrument that the federal government can use to nullify the efficiency effects of fiscal externalities. In the case of identical regions, Boadway and Keen (1996) showed that, in a simple model with distortionary labor taxation, the vertical fiscal gap can be designed to

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7 This presumes that an increase in the regional tax rate reduces the federal tax base. As Dahlby and Wilson (2003) point out, this need not be the case. If the labor income tax base is pre-tax income, a wage tax can cause the pre-tax wage to rise, which increases the federal tax base and federal revenue.
achieve a second-best federal optimum in the presence of vertical fiscal externalities. When regions differ in their fiscal capacities, things are a bit more complicated since horizontal balance must be addressed as well. As Smart (1998) argued, an equalization system that bases federal-regional transfers on the so-called representative tax system approach will largely sterilize horizontal fiscal externalities as an influence on regional behavior. Under the representative tax system approach (used in Australia and Canada), a region’s revenue-raising capacity for a given tax base is measured by the per capita revenues it would raise by applying national average regional tax rates to the given base. Equalization entitlements are then based on differences between a region’s per capita revenue-raising capacity and the national average. Given that, the tax loss arising from changes in the tax base resulting from an increase in a region’s tax rate will be approximately offset by an increase in equalization transfers (and exactly offset if the region’s tax rate is the same as the national average). Thus, not only will the horizontal externality be undone, so will the part of the MCPF that is due to a variable regional tax base. That is, the perceived MCPF will be too low. At the same time, the vertical fiscal externality will remain to a large extent intact. Bucovetsky and Smart (2006) show that in principle an equalization system can be defined, at least for a given tax base, so that the joint effect of vertical and horizontal fiscal externalities are undone. Such a system might not be viable in practice since a different equalization system would have to be defined for each regional tax base. Moreover, the purpose of equalization transfers is much broader than simply undoing the inefficiencies due to fiscal externalities. More generally, combining matching transfers with equalization can be used to achieve a second-best optimum as long as there are no restrictions on the use of those policy instruments, as Sato (2000) shows.

The literature on federal-regional transfers has also emphasized a related role of equalization, which is to provide insurance against external shocks of various sorts including those affecting productivity, preferences and costs. The empirical role of transfers in stabilizing regions against shocks has been studied by Asdrubali, Sørenson and Yoshia (1996), von Hagen and Hammond (1998), and Méliot and Zumer (2002), and the literature has been surveyed by von Hagen (2007). The theoretical literature has emphasized adverse selection (Bordignon, Manasse and Tabellini, 2001), moral hazard (Persson and Tabellini,
1996b) and redistribution (Persson and Tabellini, 1996a) and has been surveyed by Lockwood (1999). In this literature, optimality is typically violated by asymmetric information or political economy considerations. Neither vertical nor horizontal fiscal externalities play a role since there are typically no distortionary taxes or mobility. Our notion of fiscal imbalance involves how shocks, either permanent or temporary, affect fiscal externalities.

The purpose of this paper is twofold. The first is to study how mobility affects the size of the optimal fiscal gap that would be achieved by a federation of heterogeneous regions when a central government has sufficient policy instruments to implement the second-best outcome. The second is to study how fiscal imbalance is affected by both vertical and horizontal externalities in institutional settings where some policy restrictions apply. We do so in a simple model where fiscal imbalance results from shocks experienced by the regions of a federation, and where the imbalance reflects purely efficiency considerations. The basic model we use is an extension of one developed in Boadway and Tremblay (2006) to formalize the notion of fiscal imbalance. In that model, a vertical fiscal externality precluded the federal government from making sufficient transfers to achieve fiscal balance among the federal government and the regions. In this paper, we allow for mobility of taxpayers across regions to determine whether that diminishes or worsens fiscal imbalance.

The meaning of fiscal imbalance in this context is a particular one and deals mainly with the revenue side of government budgets. Expenditure responsibilities for federal and regional governments are taken as given: the federal government provides a national public good while the regions provide a regional public good for which there are no spillover benefits. Levels of public goods are chosen before regions are affected by shocks, which may be asymmetric. It is left to the federal and regional tax systems combined with federal-regional transfers to finance the public goods. The issue is how to design the federal tax and transfer system to achieve an efficient allocation of resources in the federation, given that the tax base (labor income) is both variable and mobile.

2 A Model of Fiscal Imbalance with Mobility
The model we use adapts the model of fiscal imbalance developed in Boadway and Tremblay (2006) to a setting with mobile taxpayers. The attachment-to-home version of mobility
introduced by Mansoorian and Myers (1993) is used to capture household mobility in a way that generates an interior solution for the allocation of population across regions. To simplify matters, we assume there are only two regions referred to as the poor region P and the rich region R, where the meaning of poor and rich will be defined below. Our notation convention is to denote variables in P by lowercase Roman letters, those in R by lowercase Roman letters with a bar, and federal variables by uppercase Roman letters.

There is a continuum of households with a total population normalized to unity for the federation as a whole. Households are identical except for an attachment-to-home parameter denoted $a$. They are distributed uniformly over $a \in [0, 1]$, so there is one household of each type $a$. Utility consists of two components, one involving an identical utility function in goods and leisure and the other an intrinsic attachment-to-home component. For a type $-a$ household, utility in region P is $c - h(y) + b(g) + B(G) + 1 - a$, while in region R it is $\bar{c} - h(\bar{y}) + b(\bar{g}) + B(G) + a$, where $c$ is consumption, $y$ is output endogenously supplied and $g$ is a regional public good in P, and similarly for $\bar{c}$, $\bar{y}$ and $\bar{g}$. The federal government supplies a national public good $G$. The function $h(\cdot)$, which refers to the disutility of supplying output, is increasing and strictly convex, while $b(\cdot)$ and $B(\cdot)$ are increasing and strictly concave.

Household $a$ will choose to reside in the region where the highest utility is achieved. We can identify the marginal household, say household $\tilde{a}$, as the one who is just indifferent between the two regions, which gives the migration equilibrium condition:

$$c - h(y) + b(g) + 1 - \tilde{a} = \bar{c} - h(\bar{y}) + b(\bar{g}) + \tilde{a}$$ (1)

Households with $a < \tilde{a}$ will choose P, while those with $a > \tilde{a}$ will choose R. Thus, in equilibrium the population in P is $\tilde{a}$, while that in R is $1 - \tilde{a}$.

Production consists not only of output endogenously chosen by households, but also by exogenous components that are region-specific. There is an exogenous and fixed component.
of per person production in the two regions denoted $x$ and $\pi$, where $\pi > x$. As well, there is an exogenous stochastic component denoted $z$ and $\bar{z}$, where $z, \bar{z} \in \{\varepsilon, -\varepsilon\}$. Thus, each region can have a good shock $\varepsilon$, denoted $h$, or a bad shock $-\varepsilon$, denoted $\ell$. There are four states of nature $k \in \{hh, \ell\ell, h\ell, \ell h\}$, where the first element in each state of nature refers to $P$ and the second to $R$, with the probabilities of each state being denoted $p^k$, with $\sum p^k = 1$. We make no assumptions about the distribution of states. However, for ease of exposition, we assume that the shocks are small enough relative to the fixed components $x$ and $\pi$ such that the aggregate shocks are always higher in $R$ than in $P$, that is, $\pi + \bar{z}^k > x + z^k$ in all states of nature. This means that region $P$ is always less productive than region $R$ regardless of the state of nature. The analysis extends in a straightforward way to the case where either region can be poor depending on the shock.

Governments finance their expenditures by taxes on output. Region $P$ imposes a tax of $t^k$ in state of nature $k$ while $R$ imposes $\bar{t}^k$. The federal government imposes a tax $T^k$ on output in both region $P$ and $R$. As well, the federal government makes unconditional transfers of $S^k$ and $\bar{S}^k$ to the two regions. As we shall see, taxes and transfers are state-specific, while public goods are the same for all states of nature. Moreover, population is state-specific, and the marginal household is denoted $a^k$. The budget constraints of the federal government and the regions in state of nature $k$ can be written:

$$T^k a^k(y^k + x + z^k) + T^k(1 - a^k)(\bar{y}^k + \pi + \bar{z}^k) = G + S^k + \bar{S}^k$$  \hspace{1cm} (2)

$$t^k a^k(y^k + x + z^k) + S^k = g$$  \hspace{1cm} (3)

$$\bar{t}^k(1 - a^k)(\bar{y}^k + \bar{\pi} + \bar{z}^k) + \bar{S}^k = \bar{g}$$  \hspace{1cm} (4)

In this context where independent decisions are being made by the federal government, the regional governments and households, the timing of decisions is important. We explore some consequences of different timings in what follows. The benchmark case used for comparison is what we refer to as the full commitment case where the federal government moves before the regions. The timing for this case consists of the following stages:

1. The federal government chooses $G, S^k, \bar{S}^k, T^k$.
2. The regions simultaneously choose $g, \bar{g}$. 


3. Nature chooses shocks $z^k, \bar{z}^k$.
4. The regions choose $t^k, \bar{t}^k$ to balance their budgets.
5. Households choose their region of residence.
6. Households in each region choose outputs $y^k, \bar{y}^k$.

The equilibrium outcomes are assumed to be subgame perfect. At each of the above stages, agents take as given outcomes from the previous stages and anticipate how their decisions will affect subsequent ones. As usual, we solve using backward induction. In what follows, household decisions will always follow government ones.\footnote{In fact, this is not an innocuous assumption. As Mitsui and Sato (2001) have shown, allowing households to migrate before governments choose public goods’ spending can have dramatic effects on outcomes.}

We begin by describing household decisions before turning to government ones.

### 3 Household Behavior

Consider a household in region P choosing how much output $y^k$ to supply in state $k$. Because of the quasilinearity of preferences, their choice of $y^k$ is not affected by $a, g$ or $G$. Each household regardless of their type $a$ solves the following problem:

$$
\max_{\{c^k,y^k\}} c^k - h(y^k) \quad \text{st} \quad c^k = (1 - t^k - T^k)(y^k + x + z^k)
$$

The first-order conditions reduce to $h'(y^k) = 1 - t^k - T^k$, which yields the output supply function $y^k(1 - t^k - T^k)$, with $y^k' > 0$. The value function for this problem is defined to be $v(t^k + T^k, x + z^k)$, and the envelope theorem yields:

$$
\nu_{t}^k = \nu_{T}^k = -(y^k + x + z^k), \quad \nu_{z}^k = (1 - t^k - T^k)
$$

where subscripts indicate partial derivatives.

In region R, the analogous value function is $v(t^k + T^k, \bar{x} + \bar{z}^k)$, with the envelope results denoted $\nu_{t}^k = \nu_{T}^k = -(\bar{y}^k + \bar{x} + \bar{z}^k), \quad \nu_{z}^k = (1 - \bar{t}^k - \bar{T}^k)$. Together with the value function for region P, we can rewrite the migration equilibrium condition in state $k$ as:\footnote{This assumes that the shocks $z^k, \bar{z}^k$ and region-specific productive capacities $x, \bar{x}$ apply to migrants as well.}

$$
v(t^k + T^k, x + z^k) + b(g) + 1 - a^k = v(t^k + T^k, \bar{x} + \bar{z}^k) + b(\bar{g}) + a^k
$$
In our setting, there will be more residents in region R in all states, so \( a^k < 1/2 \), although that is of no real consequence for our results.

4 The Second-Best Optimum

A natural benchmark case to consider is a planning problem in which private and public goods in each region can be chosen by a national social planner. To make the planning problem comparable to the decentralized ones considered later, we assume that the planner is subject to three sorts of constraints. The first one is a resource constraint that limits the quantities of private and public goods. The second two involve household decisions, which are assumed to be beyond the dictates of the planner. The first of these is the labor supply decision of households, and the second is the choice of region of residence. Although the planner cannot control these directly, it can influence them indirectly through labor taxes and public goods in each region (and in each state of nature). The solution to the planning problem yields the second-best optimum, given that distorting taxes cannot be avoided.

We begin by setting out the planning problem and characterizing the second-best optimal allocation. Then, we define and identify the optimal fiscal gap implicit in the second best optimum. Finally, we consider the effect of migration on the optimal fiscal gap.

The Planning Problem

Suppose that the planner provides national and regional public goods out of a national budget in each state of nature financed by taxes on household in the two regions, denoted \( \tau^k \) and \( \overline{\tau}^k \). The planner’s budget constraint in state \( k \) is:

\[
a^k \tau^k \cdot \left( y^k (1 - \tau^k) + x + z^k \right) + (1 - a^k) \overline{\tau}^k \cdot \left( \overline{y}^k (1 - \overline{\tau}^k) + \overline{x} + \overline{z}^k \right) = G + g + \overline{g}
\]

where the output supply functions \( y^k (1 - \tau^k) \) and \( \overline{y}^k (1 - \overline{\tau}^k) \) have the same properties as above, and where \( a^k \) is given by the migration equilibrium condition analogous to (6):

\[
v(\tau^k, x + z^k) + b(g) + 1 - a^k = v(\overline{\tau}^k, \overline{x} + \overline{z}^k) + b(\overline{g}) + a^k
\]

The objective function of the planner is taken to be the sum of expected utilities, taking account of the attachment-to-home component \( a^k \).\(^{11}\)

\(^{11}\) In Mansoorian and Myers (1993) and Sato (2000), the planner neglects the utility associated
τ^k to maximize the sum of expected utilities subject to the budget constraint, where \( a^k \) is determined by the migration equilibrium condition. It is convenient to allow the planner to use \( a^k \) in each state of nature as artificial control variables and to include the migration equilibrium conditions as constraints. The Lagrangian function for the planning problem denoted \( P \) is then:

\[
L = \sum_k p^k \left[ a^k \left( v(\tau^k, x + z^k) + b(g) + B(G) \right) + a^k - \int_0^{a^k} n^k dn^k \right] + \sum_k p^k \Phi^k \left[ v(\tau^k, x + z^k) + b(g) - v(\tau^k, x + z^k) - b(g) - 2a^k + 1 \right]
\]

\[
+ \sum_k p^k A^k \left[ a^k \cdot (y^k(\cdot) + x + z^k) + (1 - a^k) \tau^k \cdot (g^k(\cdot) + x + z^k) - G - g - \bar{g} \right]
\]

The first-order conditions are:

\[
G : \quad \sum_k p^k B'(G) - \sum_k p^k A^k = 0
\]

\[
g : \quad \sum_k p^k (a^k + \Phi^k) b'(g) - \sum_k p^k A^k = 0
\]

\[
\bar{g} : \quad \sum_k p^k (1 - a^k - \Phi^k) b'(\bar{g}) - \sum_k p^k A^k = 0
\]

\[
\tau^k : \quad p^k (a^k + \Phi^k) v^k_e + p^k A^k a^k \left( y^k + x + z^k - \tau^k y^k' \right) = 0 \quad \forall k
\]

\[
\tau^k : \quad p^k (1 - a^k - \Phi^k) \tau^k + p^k A^k (1 - a^k) \left( g^k + x + z^k - \tau^k \bar{g}^k \right) = 0 \quad \forall k
\]

\[
a^k : \quad p^k \left( v(\tau^k, x + z^k) + b(g) + 1 - a^k - v(\tau^k, x + z^k) - b(g) - a^k \right) - p^k 2\Phi^k + p^k A^k \left( \tau^k \cdot (y^k(\cdot) + x + z^k) - \tau^k \cdot (g^k(\cdot) + x + z^k) \right) = 0 \quad \forall k
\]

with attachment to home as a component of social welfare. Excluding attachment to home in social welfare complicates the analysis slightly, but similar results are obtained.
The interpretation of these conditions is as follows. Begin with the condition on $a_k$. It involves the shadow price of the migration constraint $\Phi_k$. The sign of $\Phi_k$ can be inferred by considering the hypothetical case in which the planner is not constrained by household mobility and could dictate where persons reside. The mobility constraint would not apply, and the first-order condition for the choice of $a_k$, which is equivalent to choosing the number of residents in each region, can be written:

$$v(\tau_k, x + z^k) + b(g) + 1 - a_k - v(\bar{\tau}_k, \bar{x} + \bar{z}^k) - b(\bar{y}) - a_k = \Lambda_k(\bar{t}_r^k - t_r^k)$$

where $t_r^k$ and $\bar{t}_r^k$ are per capita tax revenues in the two regions. Suppose that $\bar{t}_r^k > t_r^k$ for all states $k$, which is reasonable since region R always has higher exogenous income than region P. That implies that the unconstrained planner would prefer to allocate persons such that $v(\tau_k, x + z^k) + b(g) + 1 - a_k > v(\bar{\tau}_k, \bar{x} + \bar{z}^k) + b(\bar{y}) + a_k$ for the marginal person, which violates the migration equilibrium condition. The interpretation here is that per capita tax revenues are like a fiscal externality along the lines of Buchanan and Goetz (1972) or Flatters, Henderson and Mieszkowski (1974). The benefit of a new migrant to the rest of the community is the additional tax revenue the migrant produces to help finance the regional public good. If tax revenues per capita are not equal (being determined by the choice of tax rates to equalize the MCPF), utilities should not be equalized between regions. That is, the net revenue benefit from moving one person from P to R in state of nature $k$ is $\Lambda_k\left(\bar{\tau}_k \cdot (\bar{y}_k(\cdot) + \bar{x} + \bar{z}^k) - \tau_k \cdot (y_k(\cdot) + x + z^k)\right)$. Since this is positive, one should be willing to move persons until utility in R is lower than in P to offset it. Alternatively, $v(\tau_k, x + z^k) + b(g) + \Lambda_k\tau_k \cdot (y_k(\cdot) + x + z^k) + 1 - a_k$ is the social value of having one more person in P: the utility of the person moving plus the extra revenue they create to finance the public good.\(^{12}\)

If the planner must abide by the migration equilibrium constraint, which we are assuming, the migration constraint will binding. In particular, since the planner would prefer that utility for the marginal person be higher in P than in R, we have $\Phi_k < 0$ for

\(^{12}\) This finding that the migration equilibrium constraint is binding is a common finding in models of residential choice. It was first pointed out by Mirrlees (1972) in the case of urban migration and discussed by Boadway, Cuff and Marchand (2002) for an interregional model of equalization transfers.
all \( k \). This is consistent with the first-order condition on \( a^k \) which, using the migration equilibrium condition (6), can be written as:

\[
2\Phi^k = \Lambda^k \left( \tau^k \cdot (y^k + x + z^k) - \bar{\pi}^k \cdot (\bar{y}^k + \bar{x} + \bar{z}^k) \right) = \Lambda^k (\tau^k - \bar{\tau}^k) < 0
\]

From the condition on \( G \), we obtain \( B'(G) = \sum p^k A^k \), which can be interpreted as a quasi-Samuelson condition. The aggregate marginal benefit of \( G \), given that national population is normalized to unity is \( B'(G) \). Given that \( G \) must be chosen before the state of nature is revealed, the marginal benefit is set equal to the expected value of \( A^k \), that is, the expected value of the marginal cost of funds to the government.

From \( g \) and \( \bar{g} \), we have \( \sum p^k (a^k + \Phi^k) b'(g) = \sum p^k (1 - a^k - \Phi^k) b'(\bar{g}) = \sum p^k A^k \). Given that \( a^k \) and \( 1 - a^k \) are the populations in the two regions, the quasi-Samuelson conditions for regional public goods would apply if \( \Phi^k = 0 \) for all \( k \). Since \( \Phi^k < 0 \), \( g \) tends to be under-provided and \( \bar{g} \) over-provided. This is consistent with migration constraining the planner to have too few persons in region R relative to P. Note that if the regions were ex ante identical so \( x = \bar{x} \), it would be the case that \( g = \bar{g} \), so \( b'(g) = b'(\bar{g}) \). From the conditions on \( g \) and \( \bar{g} \), we obtain \( \sum p^k (a^k + \Phi^k) = \sum p^k (1 - a^k - \Phi^k) \), which implies that \( \sum p^k (a^k + \Phi^k) = 1/2 \), so the condition become \( b'(g)/2 = b'(\bar{g})/2 = \sum p^k \lambda^k \), which are the quasi-Samuelson conditions given that expected regional populations here are 1/2 each.

Finally, from \( \tau^k \) and \( \bar{\tau}^k \) and using the envelope conditions, we obtain:

\[
\Lambda^k = \frac{a^k + \Phi^k}{a^k} \frac{y^k + x + z^k}{y^k + x + z^k - \tau^k y^k'} = \frac{1 - a^k - \Phi^k}{1 - a^k} \frac{\bar{y}^k + \bar{x} + \bar{z}^k}{\bar{y}^k + \bar{x} + \bar{z}^k - \bar{\tau}^k \bar{y}^k'}
\]

The term \( (y^k + x + z^k)/(y^k + x + z^k - \tau^k y^k') \) can be interpreted as the marginal cost of public funds (MCPF\(^k\)) in state of nature \( k \) for region P, and similarly for region R, where we denote by MCPF\(^k\) the marginal cost of public funds in region R. Given that \( \Phi^k < 0 \), taxes in the two regions will be set such that:

\[
\text{MCPF}^k = \frac{y^k + x + z^k}{y^k + x + z^k - \tau^k y^k'} > \Lambda^k > \frac{\bar{y}^k + \bar{x} + \bar{z}^k}{\bar{y}^k + \bar{x} + \bar{z}^k - \bar{\tau}^k \bar{y}^k'} = \text{MCPF}^k
\]

If the marginal cost of public funds is increasing in the tax rate, as we reasonably assume to be the case, that implies that \( \tau^k \) is higher than in the full planning solution and \( \bar{\tau}^k \)
lower. This attracts more migrants to R to offset the tax externality. Note also that if there were no migration, the migration constraint would not appear, and the planning solution would involve equalizing the MCPF between the two regions in each state of the world. We use the no-migration case as a point of comparison in what follows.

The Optimal Fiscal Gap
It is useful for comparison purposes to characterize the decentralized equivalent of the planning solution, that is, where regional governments provide regional public goods \( (g \text{ and } \overline{g}) \), the federal government provides the national public good \( (G) \), both levels of government have access to a tax on income \( (t^k, T^k) \), and there is a system of federal-regional transfers \( (S^k, \overline{S}^k) \). Imagine that regional and federal governments behave cooperatively so that all externalities are internalized. In this setting, where vertical and horizontal externalities are irrelevant, the vertical fiscal gap is indeterminate, as discussed in Boadway and Tremblay (2006). Regional and federal tax rates are set such that \( t^k + T^k = \tau^k \) and \( t^{\overline{k}} + T^{\overline{k}} = \pi^{\overline{k}} \), so the planning tax rate is achieved. Transfers \( S^k \text{ and } \overline{S}^k \) are set such that regions are able to finance the planning levels of \( g \text{ and } \overline{g} \), given their own tax revenues. In any state of nature, the relative size of optimal transfers \( S^k, \overline{S}^k \) is determinate but not the absolute level: an increase in both transfers combined with an increase in federal tax rates and a decrease in regional tax rates will leave the outcome unaffected.

To pin down the vertical fiscal gap in a way that allows for a comparison with the following sections, we assume that the planner opts for the minimum level of transfers in each state of nature consistent with them being non-negative. This implies that \( S^k, \overline{S}^k \geq 0 \), with at least one equality. In our benchmark setting, where P always has the lowest exogenous per capita income, region P will always have positive transfers and region R will have none, so \( S^k > 0, \overline{S}^k = 0 \) for all \( k \). This simplifies our analysis in what follows. More generally, in any state of nature, one region will receive a transfer and the other will not, and the optimal fiscal gap will be region-specific. If there were more than two regions, all but one would receive positive transfers of varying size.

The Effect of Migration on the Optimal Fiscal Gap
It is useful to consider the effect of migration on the size of the fiscal gap, which surprisingly
turns out to be somewhat ambiguous. To focus attention on the reason for the ambiguity, consider the special case in which the two regions are ex ante identical, so \( x = \bar{x} \). The arguments will apply to the more general case as well. With ex ante identical regions, both regions have the same initial populations (that is, \( \bar{a} = 1/2 \)) and in the planning optimum, \( g = \bar{g} \) so both regions require the same amount of revenues. Suppose as a benchmark that there is no mobility. In this case, the planning optimum involves equalizing the MCPF between the two regions in all states of nature, with regional public goods being chosen such that the marginal benefit to regional residents equals the expected value of the MCPF across states. With asymmetric shocks, the region with the good shock (which could be either region) has the highest tax rate and the highest revenue. The low-shock region obtains a positive transfer equal to the optimal fiscal gap.

Starting in this benchmark optimum, imagine now that migration occurs. The first problem is that it is ambiguous in which region per capita utility is highest. The good-shock region has a higher tax rate, so supplies less discretionary income \( y \), and may have higher or lower consumption. Therefore, it it not clear whether migration goes from the bad- to the good-shock region, or vice versa. Suppose, reasonably, that utility before migration is higher in the region with the good shock. Then, migration goes from the bad- to the good-shock region and has the following effects on the fiscal gap. First, given the no-migration tax rates, tax revenues fall in the bad-shock region and rise in the good-shock region. Second, since the aggregate national tax base rises with migration, so do aggregate tax revenues. Thus, the federal government can reduce tax rates. If tax rates were reduced to keep the MCPF the same in both regions, tax revenue in the bad-shock region would fall. On both counts, tax revenue falls in the bad-shock region, so a higher transfer must be made to enable it to finance the no-migration level of the regional public good. Further, since tax rates have fallen, the level of regional public goods chosen ex ante would rise and this would contribute to an additional source of increase in transfers to the bad-shock region. Finally, as we have seen, the planner would want to move away from equalizing the MCPF in the two regions. Given that per capita tax revenues are higher in the good-shock region, the planner would encourage migration by increasing the tax rate in the bad-shock region. That would both restrict migration and increase tax
revenues in the region, thereby reducing the need for transfers. Overall, we presume that
the migration combined with the adjustment of regional tax rates would reduce the tax
take in the bad-shock region and cause the optimal fiscal gap to rise. However, as this
discussion indicates, even thought this is reasonable, it is by no means unambiguous.

5 The Non-Cooperative Outcome with Federal Commitment
In this section, we assume that the federal government can commit to its policies announced
in Stage 1 before the regions act. Its announcement consists of \( G, T^k, S^k \) and \( \overline{S}^k \), but
these must be consistent with its budget, given regional choices. It suffices to let the
federal government choose \( G \) and \( T^k \), with \( S^k \) and \( \overline{S}^k \) being determined endogenously in
each state to balance its budget. Given the federal government’s announced policies, the
regions choose \( g \) and \( g' \) to maximize the sum of utilities in their respective regions subject
to their budget constraints and taking as given the policies announced by the other region.
They anticipate how their policies affect fiscal transfers \( S^k \) and \( \overline{S}^k \) and also how they affect
migration equilibrium \( a^k \). As mentioned, we focus on the benchmark case in which \( S^k > 0 \)
and \( \overline{S}^k = 0 \) for all states. This minimizes the size of the federal tax rate, and therefore the
vertical fiscal externality. Later we discuss whether this is the most likely scenario.

Given that \( S^k > 0 \) and \( \overline{S}^k = 0 \), the federal budget in state \( k \) may be written:

\[
S^k = T^k a^k \left( y^k (t^k + T^k, x + z^k) + x + z^k \right) + T^k (1 - a^k) \left( \overline{y}^k (T^k + t^k, \overline{x} + \overline{z}^k) + \overline{x} + \overline{z}^k \right) - G
\]

The effect that policies have on transfers are then as follows:

\[
\frac{\partial S^k}{\partial t^k} = -T^k a^k y^k' < 0, \quad \frac{\partial S^k}{\partial T^k} = -T^k (1 - a^k)\overline{y}^k' < 0
\]

\[
\frac{\partial S^k}{\partial a^k} = T^k (y^k + x + z^k - T^k y^k') + (1 - a^k) (\overline{y}^k + \overline{x} + \overline{z}^k - \overline{T}^k \overline{y}^k')
\]

\[
\frac{\partial S^k}{\partial G} = -1
\]

To solve for the equilibrium in this case, we use backward induction and start with
the region’s choices.
The Problem of Region P

Region P chooses \( g \) to maximize the total utility of its residents, anticipating its tax rate determined at a later stage by its budget and its population determined by migration, and anticipating the effect of its choice on its transfer \( S^k \). Although \( t^k \) and \( a^k \) are determined endogenously, it is convenient for analytical purposes to let region P use \( t^k \) and \( a^k \) as artificial control variables, adding its budget constraint and migration equilibrium as constraints. The Lagrangian expression for region’s problem, denoted \( C \), is:

\[
L = \sum p^k \left[ a^k \left( v(t^k + T^k, x + z^k) + b(g) + B(G) \right) + a^k - \int_0^{a^k} n^k dn^k \right] + \sum p^k \lambda^k \left[ t^k a^k (y^k(t^k + T^k, x + z^k) + x + z^k) + S^k(\cdot) - g \right] + \sum p^k \mu^k \left[ v(t^k + T^k, x + z^k) + b(g) - v(t^k + T^k, x + z^k) - b(\overline{y}) + 1 - 2a^k \right] \]

where the properties of \( S^k(\cdot) \) are given by (7). The first-order conditions are (using the envelope conditions):

\( g : \) \( \sum p^k (a^k + \mu^k)b(g) - \sum p^k \lambda^k = 0 \)

\( t^k : \) \( -(a^k + \mu^k)(y^k + x + z^k) + \lambda^k a^k (y^k + x + z^k - t^k y^k) + \lambda^k \frac{\partial S^k}{\partial t^k} = 0 \)

\( a^k : v(t^k + T^k, x + z^k) + b(g) + B(G) + 1 - a^k + \lambda^k t^k \cdot (y^k + x + z^k) + \lambda^k \frac{\partial S^k}{\partial a^k} - 2\mu^k = 0 \)

where the last two apply for all states \( k \).

The interpretation of these is as follows. The condition on \( a^k \) gives

\[
2\mu^k = v(t^k + T^k, x + z^k) + b(g) + B(G) + 1 - a^k + \lambda^k t^k \cdot (y^k + x + z^k) + \lambda^k T^k (y^k + x + z^k - \overline{y} - \overline{x} - \overline{z})
\]

The righthand side consists of two components. The first is the value of having an additional migrant, consisting of the utility of that migrant, plus the value of the tax revenue
that the migrant adds to the region (the fiscal externality). This term is positive, and reflects the incentive to the region of attracting residents. The second component involving $T^k$ is the reduction in federal transfers from having another migrant. It reflects the fact that federal tax revenues fall when a migrant moves from R to P, and therefore so does the size of the fiscal transfer $S^k$. Provided that the difference in tax bases is not too large and/or that the federal tax rate is not too high relative to the regional tax rate, $\mu^k$ will be positive. That is, region P will have an incentive to attract residents from R. We assume reasonably that this is the case.

From the first-order condition for $t^k$ in state $k$, we have:

$$\lambda^k = \frac{a^k + \mu^k}{a^k} \frac{y^k + x + z^k}{y^k + x + z^k - (T^k + t^k)y^{k'}}$$

(8)

This is region P’s perceived MCPF. It consists of the social value of the MCPF multiplied by $(a^k + \mu^k)/a^k$. Two observations are relevant. First, a horizontal externality is reflected by the presence of $\mu^k$, which tends to increase the region’s perceived MCPF. That is because an increase in the region’s tax rate discourages migrants. Second, there is no vertical fiscal externality for region P. That is because of the term $\partial S^k/\partial t^k$ in the first-order condition for $t^k$, which induces region P to take account of the effect of its tax change on the federal budget, given that the latter ends up reducing the transfer to P. This absence of a vertical fiscal externality is due to the fact that only one region P receives a transfer. A vertical externality would appear in more realistic scenarios such as where P did not receive a transfer in all states of nature, or if there were more transfer-receiving regions.

Combining (8) for the state’s perceived MCPF with the condition on $g$, we obtain:

$$\sum p^k a^k \frac{a^k + \mu^k}{a^k} a^k b'(g) = \sum p^k a^k \frac{a^k + \mu^k}{a^k} \frac{y^k + x + z^k}{y^k + x + z^k - (T^k + t^k)y^{k'}}$$

(9)

This expression would give the quasi-Samuelson social optimality condition if $\mu^k = 0$. Since $\mu^k > 0$, that condition will not be precisely satisfied. However, since $\mu^k$ appears in a similar way on both sides of (9), there is no general tendency for $g$ to be set too high or too low. Intuitively, a higher $g$ will attract more migrants, but it will also require a higher tax rate which will have the opposite effect. The two more or less offset. Overall, there is
no general tendency for the poor region to use expenditures on \( g \) too aggressively, but it will end up with too low a tax rate.

The solution to problem \( C \) gives a value function representing total utility in \( P \), which we denote as \( w(G, T) \), where \( T \) is the vector of federal tax rates \( T^k \) in the four states of nature. Using the envelope theorem, differentiating the Lagrangian for region \( P \)'s problem \( C \) yields the following:

\[
\frac{\partial w(G, T)}{\partial G} = \sum p^k a^k B'(G) + \sum p^k \lambda^k \frac{\partial S^k}{\partial G} = \sum p^k a^k B'(G) - \sum p^k \lambda^k
\]  

\[
\frac{\partial w(G, T)}{\partial T^k} = -p^k (a^k + \mu^k)(y^k + x + z^k) - p^k \lambda^k t^k a^k y^k' + p^k \lambda^k \frac{\partial S^k}{\partial T^k} + p^k \mu^k(y^k + \bar{x} + \bar{z}^k)
\]

Using the first-order conditions for region \( P \)'s problem and (7), the latter may be rewritten:

\[
\frac{\partial w(G, T)}{\partial T^k} = p^k \lambda^k (1 - a^k)(\bar{y}^k + \bar{x} + \bar{z}^k - T^k \bar{y}^k') + p^k \mu^k(\bar{y}^k + \bar{x} + \bar{z}^k)
\]  

(10)

which is unambiguously positive in sign. It reflects the fact that an increase in the federal tax rate holding \( G \) constant results in a higher transfer to region \( P \).

The Problem of Region \( R \)

Region \( R \)'s problem is similar except it receives no transfer. We can write its Lagrangian expression as follows:

\[
\mathcal{L} = \sum p^k \left[ (1 - a^k) \left( v(T^k + \bar{x} + \bar{z}^k) + b(\bar{y}) + B(G) \right) + \int_\alpha^1 n^k dn^k \right] \\
+ \sum p^k \lambda^k \left[ t^k (1 - a^k)(\bar{y}^k (T^k + \bar{x} + \bar{z}^k) + \bar{x} + \bar{z}^k) - \bar{y} \right] \\
+ \sum p^k \nu^k \left[ v(T^k + \bar{x} + \bar{z}^k) + b(\bar{y}) + a^k - v(T^k, x + z^k) - b(g) + 1 - a^k \right]
\]

From the first-order conditions, we obtain:

\[
2\bar{\nu}^k = v(T^k + \bar{x} + \bar{z}^k) + b(\bar{y}) + B(G) + a^k + \lambda^k t^k \cdot (\bar{y}^k + \bar{x} + \bar{z}^k) > 0
\]

\[
\lambda^k = \frac{1 - a^k + \bar{\nu}^k}{1 - a^k} \frac{\bar{y}^k + \bar{x} + \bar{z}^k}{1 - a^k - \bar{t}^k \bar{y}^k'}
\]

\[
\sum p^k \frac{1 - a^k + \bar{\nu}^k}{1 - a^k} (1 - a^k) b'(\bar{y}) = \sum p^k \frac{1 - a^k + \bar{\nu}^k}{1 - a^k} \frac{\bar{y}^k + \bar{x} + \bar{z}^k}{\bar{y}^k + \bar{x} + \bar{z}^k - \bar{t}^k \bar{y}^k'}
\]
These differ from the case of region P by the fact that there is now a vertical fiscal externality that is not offset by a change in transfers. The first expression indicates that there is an unambiguous incentive to attract migrants both because of the addition to total utility of the migrant and because of the fiscal externality corresponding to the additional regional tax revenue generated by the migrant. As the second equation indicates, that incentive to attract migrants increases the region’s perceived MCPF by the term involving $\mu^k$, but this is to some extent offset by the vertical fiscal externality resulting from the fact that the region understates the social MCPF by ignoring the effect an increase in its tax rate has on federal revenues (i.e., by the absence of the term in $T_k$ in the denominator). On balance, the region may over- or under-estimate its MCPF. This misperception of the social MCPF then feeds into its choice of $g$ in the last equation, which induces it to either over- or under-provide $\bar{g}$ depending on the relative weight of horizontal and vertical fiscal externalities. Note that if the horizontal externality arising from migration dominates, the federal government would want to give region R a positive transfer, while maintaining the appropriate differential between transfers to the two regions. This might induce region R’s MCPF to be the optimal one, as in Bucovetsky and Smart (2006).

We define the value function for region R’s problem as $\bar{w}(G, T)$. Proceeding as in the case of region P, applying the envelope theorem to region R’s problem and using its first order conditions, the derivatives of $\bar{w}(G, T)$ can be written:

$$\frac{\partial \bar{w}(G, T)}{\partial G} = \sum p^k (1 - a^k) B'(G)$$

$$\frac{\partial \bar{w}(G, T)}{\partial T_k} = -p^k \lambda^k (1 - a^k)(\bar{y}^k + \bar{x} + \bar{z}^k) + p^k \mu^k (\bar{y}^k + x + z^k)$$

The Problem of the Federal Government

We continue to assume that federal transfers cannot be negative in any state. If this is binding in all states, which we take to be the case for expository purposes, this implies that $S^k > 0$ for region P and $\bar{S}^k = 0$ for R. It also implies a positive value of $T^k$ in all states to finance $G$ and $S^k$, and thus a positive vertical externality for region R. The federal government’s problem is simply to choose $G$ and $T^k$ to maximize the sum of utilities in P and R, where the latter are given by the value functions $w(G, T)$ and $\bar{w}(G, T)$. Since
these functions take account of migration responses, there is not need to do so again. The
problem of the federal government is therefore:

$$\max_{\{G, T^k\}} w(G, T) + \bar{w}(G, T)$$

Using the envelope theorem results (10)–(13) derived above, the first-order conditions can
be written as

$$\sum p^k a^k B'(G) - \sum p^k \lambda^k + \sum p^k (1 - a^k) B'(G) = 0$$

$$\lambda^k (1 - a^k) (\bar{y}^k + \bar{x} + \bar{z}^k - T^k \bar{y}^k) + \mu^k (\bar{y}^k + \bar{x} + \bar{z}^k) - \bar{\lambda}^k (1 - a^k) (\bar{y}^k + \bar{x} + \bar{z}^k) + \bar{\mu}^k (y^k + x + z^k) = 0$$

The first of these simplifies to $B'(G) = \sum p^k \lambda^k$. The level of provision of the federal
public good is such that its marginal benefit equals the expected marginal cost of public
funds evaluated using the MCPF in the poor region. For a given federal tax rate, an
increase in $G$ reduces the transfer to the poor region by an equal amount, and the marginal
value of the transfer to the poor region is equal to the poor region’s MCPF multiplied by
its expected population. Given that the MCPF perceived by region P overestimates the
social MCPF as mentioned, the federal government will tend to under-provide $G$ relative
to the planning optimum as a consequence of reducing its tax rate to reduce the vertical
fiscal externality.

The second equation indicates how the perceived MCPF’s differ between the two
regions, and therefore how inefficiently revenues are raised across the two regions. It can
be rewritten as follows:

$$\frac{\lambda^k}{\bar{\lambda}^k} = \frac{\bar{y}^k + \bar{x} + \bar{z}^k}{\bar{y}^k + \bar{x} + \bar{z}^k - T^k \bar{y}^k} - \frac{\mu^k (\bar{y}^k + \bar{x} + \bar{z}^k) + \bar{\mu}^k (y^k + x + z^k)}{\bar{\lambda}^k (1 - a^k) (\bar{y}^k + \bar{x} + \bar{z}^k) - T^k \bar{y}^k}$$

Consider the two components on the righthand side. The first component is greater than
unity. To interpret it, suppose there were no migration, so $\mu^k = \bar{\mu}^k = 0$ and only vertical
fiscal externalities apply. To mitigate this, the federal government reduces its tax rate
below the second-best optimal value and reduces transfers to region P. The result is that
region P’s tax rate must be high enough such that its MCPF is higher than that in region R. This might be interpreted as a negative fiscal imbalance in all states of nature since $S^k$
is below its second-best optimal value in all states. The second term, which is negative,
indicates that migration tends to reduce the MCPF of the poor region relative to that of the rich region, which requires larger federal transfers. That is, migration reduces the tendency for a negative fiscal imbalance.

In fact, as mentioned, the horizontal fiscal externality arising from migration could be large enough to offset the vertical fiscal externality in the case of region R and possibly even cause transfers to this region to be positive. In this case, the negative fiscal imbalance would disappear and the federal government could achieve the desired differential in transfers to the two regions. Still, the planning optimum would not be achieved since region P would still be affected by horizontal fiscal externalities. Presumably these could only be eliminated by making transfers in each state conditional on regional spending.

To summarize the full commitment case, it is useful to make reference to the case with no mobility. In that case, where there are no horizontal externalities, there is always a negative fiscal imbalance under full commitment. That is, the transfer to region P is lower than in the full planning solution. The reason is that the federal government wants to make its tax rate as low as possible to eliminate the vertical fiscal externality, but is limited in that regard by an inability to make transfers negative to region R. The public good in region R is too high because of the vertical fiscal externality, but too low in region P because of the negative fiscal imbalance and too low at the federal level because the federal tax rate is limited to reduce the vertical fiscal externality. Mobility moderates these results by introducing a horizontal externality that induces both regions to attract residents. This is reflected in regions perceiving their MCPFs to be higher than the socially optimal ones, which tends to cause them to decrease their public goods. This serves to offset vertical fiscal externalities that exist, at least in region R, and allows the federal government to increase its tax rate and thus the level of its transfer, that is, to reduce the negative fiscal imbalance. If the horizontal fiscal externality is strong enough, it will be desirable to make positive transfers to both regions, thereby eliminating the non-negative transfer constraint that the federal government otherwise faces. The federal government can thus achieve a more efficient allocation of revenue-raising between the two regions, but it cannot fully offset the horizontal externality in the two regions. In that sense, mobility is a good thing when the federal government can commit.
6 The Non-Cooperative Outcome with No Commitment

In this section, we assume that the federal government cannot commit to its tax and transfer policy before regions choose their levels of expenditures. We take this inability to commit into account by assuming that the federal tax and transfer policy is chosen after regional policies have been determined. The timing of decisions is as follows:

1. The federal government chooses $G$.
2. Regions simultaneously choose $g, \bar{g}$.
3. Nature chooses shocks $z^k, \bar{z}^k$.
4. The federal government chooses $S^k, \bar{S}^k, T^k$.
5. Regions choose $t^k, \bar{t}^k$ to balance their budgets.
6. Households choose their region of residence.
7. Households in each region choose outputs $y^k, \bar{y}^k$.

Again, we characterize the subgame-perfect equilibrium. Household behavior is the same as before, so we can start with the federal government’s choice of $T^k, S^k$ and $\bar{S}^k$.

The Tax and Transfer Policy of the Federal Government

At this stage, $G, g$ and $\bar{g}$ are given, and the federal government anticipates $a^k, t^k$ and $\bar{t}^k$. Since $g$ and $\bar{g}$ are given, there is no need to worry about vertical externalities, so the level of $T^k$ is not a problem. The federal government is concerned only with efficiency of raising revenues across regions and of migration. Since the state of nature has been revealed, we can consider the federal government problem for a given state $k$.

The federal government maximizes the sum of utilities subject to its budget constraint and anticipating $a^k, t^k$ and $\bar{t}^k$ from the migration constraint and the regional budget constraint. As usual, we let $a^k, t^k$ and $\bar{t}^k$ be artificial control variables and add the relevant constraints to the problem. The Lagrangian expression is:

$$\mathcal{L} = a^k \left( v(t^k + T^k, x + z^k) + b(g) + B(G) \right) + a^k - \int_0^{a^k} n^k dn^k$$

$$+ (1 - a^k) \left( v(\bar{t}^k + T^k, \bar{x} + \bar{z}^k) + b(\bar{g}) + B(G) \right) + \int_{a^k}^1 n^k dn^k$$

$$+ \Phi^k \left( v(t^k + T^k, x + z^k) + b(g) - v(\bar{t}^k + T^k, \bar{x} + \bar{z}^k) - b(\bar{g}) - 2a^k + 1 \right)$$
The first-order conditions for $S^k$, $\bar{S}^k$, $t^k$, $\bar{t}^k$ and $a^k$ are:

$$S^k : \quad \lambda^k - A^k = 0$$

$$\bar{S}^k : \quad \bar{\lambda}^k - A^k = 0$$

$$t^k : \quad -(a^k + \Phi^k)(y^k + x + z^k) + \lambda^k a^k(y^k + x + z^k - t^k y^{k'}) - A^k a^k T^k y^{k'} = 0$$

$$\bar{t}^k : \quad -(1 - a^k - \Phi^k)(\bar{y}^k + \bar{x} + \bar{z}^k) + \bar{\lambda}^k (1 - a^k)(\bar{y}^k + \bar{x} + \bar{z}^k - \bar{t}^k \bar{y}^{k'}) - A^k (1 - a^k) T^k \bar{y}^{k'} = 0$$

$$a^k : \quad \lambda^k t^k \cdot (y^k + x + z^k) - \bar{\lambda}^k \bar{t}^k \cdot (\bar{y}^k + \bar{x} + \bar{z}^k) + A^k T^k (y^k + x + z^k - \bar{y}^k - \bar{x} - \bar{z}^k) = 2 \Phi^k$$

where we have used the migration equilibrium condition in the equation for $a^k$.

From the conditions characterizing the choice of $S^k$ and $\bar{S}^k$, we immediately obtain $\lambda^k = \bar{\lambda}^k = A^k$. After shocks have been revealed and decisions over the provision of regional public goods have been made, the optimal policy for the federal government is to equalize the shadow price of government revenue from all budgets. Although the regions’ anticipation of this federal policy will distort their behavior in the previous stage, the federal government cannot commit to do otherwise.

From $a^k$, given $\lambda^k = \bar{\lambda}^k = A^k$ and using the migration constraint, we obtain:

$$A(t^k + T^k)(y^k + x + z^k) - A(\bar{t}^k + T^k)(\bar{y}^k + \bar{x} + \bar{z}^k) = 2 \Phi^k$$

This condition has the same form as in the planning solution. The migration constraint is binding in the problem of the federal government given that per capita tax revenues are not equal between the two regions.

From $t^k$ and $\bar{t}^k$, using $\lambda^k = \bar{\lambda}^k = A^k$, we obtain:

$$\frac{a^k + \Phi^k}{a^k} \frac{y^k + x + z^k}{(t^k + T^k) y^{k'}} = \lambda^k = \bar{\lambda}^k = \frac{1 - a^k - \Phi^k}{1 - a^k} \frac{\bar{y}^k + \bar{x} + \bar{z}^k}{(\bar{t}^k + T^k) \bar{y}^{k'}}$$

(15)
Thus, the MCPFs are adjusted to take account of migration, as in the planning solution.

The Problem of Region P
Region P chooses $g$ to maximize expected utility, taking $G$ as given, and anticipating the federal choice of tax rate and transfer in the next stage as well as the region’s own tax rate. We take that into account by treating $S^k, T^k, t^k$ and $a^k$ as artificial control variables and adding as constraints the regional and federal budget constraints, the migration constraint and condition (15) which characterize the federal government’s policy in the next stage. The problem is:

$$L = \sum p^k \left( a^k \left( v(t^k + T^k, x + z^k) + b(g) + B(G) \right) + a^k - \int_{0}^{a^k} n^k dn^k \right)$$

$$+ \sum p^k \delta^k \left( t^k a^k (y^k(\cdot) + x + z^k) + S^k - g \right)$$

$$+ \sum p^k \gamma^k \left( a^k T^k \cdot (y^k(\cdot) + x + z^k) + (1 - a^k) T^k \cdot (\bar{y}^k(\cdot) + \bar{x} + \bar{z}^k) - G - S^k - \bar{S}^k \right)$$

$$+ \sum p^k \omega^k \left( v(t^k + T^k, x + z^k) + b(g) - v(T^k, x + z^k) - b(\bar{g}) + 1 - 2a^k \right)$$

$$+ \sum p^k \eta^k \left( \lambda^k(\cdot) - \bar{\lambda}^k(\cdot) \right)$$

The first-order conditions for $g, t^k$ and $S^k$ are:

$g$:

$$\sum p^k a^k b'(g) - \sum p^k \delta^k + \sum p^k \omega^k b'(g) = 0$$

$t^k$:

$$- a^k (y^k(\cdot) + x + z^k) + \delta^k \left( a^k (y^k(\cdot) + x + z^k) - t^k a^k y^k' \right) - \gamma^k a^k T^k y^k'$$

$$- \omega^k (y^k(\cdot) + x + z^k) + \eta^k \left( \frac{\partial \lambda^k}{\partial t^k} - \frac{\partial \bar{\lambda}^k}{\partial t^k} \right) = 0$$

$S^k$:

$$\delta^k - \gamma^k = 0$$

We can rewrite the condition for $g$ as:

$$\sum p^k (a^k + \omega^k) b'(g) = \sum p^k \delta^k$$
which has the same form as in the planning solution. Using the conditions for \( t^k \) and \( S^k \), we obtain:

\[
\delta^k = \frac{y^k(\cdot) + x + z^k}{y^k(\cdot) + x + z^k - (t^k + T^k)y^k} \left[ \frac{a^k + \omega^k}{a^k} + \frac{\eta^k}{a^k(y^k(\cdot) + x + z^k)} \left( \frac{\partial \lambda^k}{\partial t^k} - \frac{\partial \lambda^k}{\partial t^k} \right) \right] \quad (16)
\]

The MCPF perceived by region P is distorted by two effects. First, the MCPF is multiplied by \((a^k + \omega^k)/a^k\) (as in the full commitment case), which reflects the horizontal externality associated with migration. This effect tends to increase the MCPF perceived by region P. The second term in the square brackets on the righthand side of (16) reflects the incentive of the regional government to over-spend in order to attract a larger transfer from the federal government. From (15), and using (14) as well as \( \lambda(\cdot) = \overline{\lambda}(\cdot) \), we can verify that \( \partial \lambda^k / \partial t^k > 0 \) and \( \partial \lambda^k / \partial t^k < 0 \). Therefore, the last term in equation (16) is negative, which tends to lower the MCPF perceived by region P.

The problem of region R is analog to that of region P and its perceived MCPF is distorted by the same effects, so there is no need to analyze the problem in detail. Even if region R receives no transfer from the federal government, increasing \( g \) will tend to reduce the transfer provided to region P, some of which is financed by federal taxes in region R. Therefore, the same incentive for over-spending will apply in region R.

Without mobility, the first distortion in the perceived MCPF described above would be absent, and both regions would necessarily under-estimate their MCPF and over-provide their public good. Given that the federal government subsequently set transfers to equalize the perceived MCPF across regions, there would be a positive fiscal imbalance in the sense that transfers in all states would tend to be larger than in the optimum. However, the horizontal externality associated with migration increases the MCPF perceived by regions, and therefore, mobility tends to reduce the size of the positive fiscal imbalance. In effect, mobility is like a precommitment device that restricts the incentive of the regions to exploit the federal government’s inability to commit to a hard budget constraint.

The Spending Decision of the Federal Government

In the first stage, the federal government chooses its level of expenditures anticipating regional spending policies and its own subsequent choice of taxes and transfers ex post in
each state of nature. Let $w(G)$ and $\overline{w}(G)$ denote the value functions of the two regions. The federal government’s choice of $G$ solves

$$\max_G w(G) + \overline{w}(G)$$

Applying the envelope theorem to the regional problems, and using $\gamma^k = \delta^k$ as well as its analog for region $R$, the first-order condition is:

$$\sum p^k a^k B'(G) - \sum p^k \delta^k + \sum p^k (1 - a^k) B'(G) - \sum p^k \gamma^k = 0$$

or

$$B'(G) = \sum p^k (\delta^k + \gamma^k)$$

The federal public good is set such that the marginal benefit equals the expected MCPF perceived by the two regional governments. Therefore, relative to the planning solution, the federal public good may be over- or under-provided, as are regional public goods.

7 Conclusions and Extensions

In this paper, we characterized the optimal fiscal gap and the fiscal imbalances that can arise in a federation where different governments are making decisions sequentially and non-cooperatively, and examined how labor mobility affects the size of both the fiscal gap and the imbalances. Under reasonable circumstances, mobility increases the size of the optimal fiscal gap entailing a higher share of revenues assigned to the federal government. In considering the effect on fiscal imbalance, our analysis distinguished between the cases where the federal government can and cannot commit to a structure of taxes and transfers to regions before the levels of regional spending are set.

In the case where the federal government can commit, the presence of a vertical fiscal externality tends to lower the MCPF perceived by regional governments, especially for the rich region, which expects to receive limited federal transfers. In the absence of mobility, this distortion induces regional over-spending. In order to reduce the size of the vertical fiscal externality, the federal government commits to relatively low tax rates and transfers, which results in a negative fiscal imbalance in the sense that transfers to poorer regions in any state of nature are lower than in the second-best optimum. The presence of labor
mobility, generating horizontal fiscal externalities, works in the opposite direction. It tends to increase the MCPF perceived by regional governments and reduce the size of the fiscal imbalance associated with the federal government’s tax and transfer policy. If the horizontal fiscal externality dominates the vertical one, the fiscal imbalance may disappear altogether, though not the externalities themselves. Achieving the second-best optimum would generally require matching transfers, as Sato (2000) shows.

In the case where the federal government cannot commit to taxes and transfers, regions tend to over-spend in order to attract larger transfers (or to reduce transfers to poor regions, in the case of rich ones). In effect, their budget constraints are not completely hard. Without mobility, regions necessarily under-estimate the MCPF and over-provide public goods. In turn, the incentive of the federal government to equalize the MCPF across regions ex post results in a positive fiscal imbalance, i.e. transfers are larger than in the second-best optimum. Again, mobility tends to reduce the size of the imbalance since the desire of regions to attract migrants increases their perceived MCPF.

As an extension to our analysis, it could be interesting to examine the impact of labor mobility in a setting where regions would face an even softer budget constraint. In the context of our model, this would correspond to a case where regions would essentially set their taxes to zero (or at very low levels) and let federal transfers, determined ex post, finance their spending. With low regional tax rates, the incentive of regions to attract migrants would tend to disappear given that the benefit of an additional migrant is essentially the regional tax revenues the migrant generates. Thus, the mitigating effect that mobility was found to have on fiscal imbalances would tend to go away under soft-budget constraints.

It might also be worth characterizing tax and transfer policies in a setting where migration decisions would be made before government policies are determined. As shown in Mitsui and Sato (2001), when migration decisions occur first and governments are unable to commit to transfer policies, individuals’ anticipation of ex-post transfers can lead to greater population concentration than in the absence of any interregional transfers. It would perhaps be interesting to examine how our results about the impact of mobility on fiscal imbalances would change in such a setting.
The broad message of this paper from a tax policy or tax assignment perspective is that mobility enhances the case for more centralized revenue-raising in a federation, while it tends to mitigate the fiscal imbalances that arise in federations that may face asymmetric shocks or that have asymmetric regions to begin with. In the Canadian case, which motivated our interest in fiscal imbalance, the federation has in recent years been subject to both substantial aggregate shocks and substantial asymmetric shocks, both of which have led to fiscal imbalances, arguably of the negative sort. At least in the case of the latter, mobility has played an important role in mediating the consequences of the shocks for fiscal balance. Nonetheless, the sheer size of the asymmetric shock, which took the form of a resource boom in selected provinces, has resulted in significant fiscal imbalance. Given the highly decentralized nature of the Canadian fiscal system, the federal government has had limited ability to respond. Perhaps encouraging federal-provincial cooperation by some form of institutional change could be helpful. On the other hand, horizontal imbalance also entails some equity concerns, and incorporating them into our analysis would be a useful next step.
References


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